

ENGAGING STUDENTS WITH SIGNIFICANT MATHEMATICAL CONTENT FROM THE SIMPSONS

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ABSTRACT: *The Simpsons* is an ideal source of fun ways to introduce important mathematical concepts, motivate students, and reduce math anxiety. We discuss examples from *The Simpsons* related to calculus, geometry, and number theory that we have incorporated into the classroom. We explore student reactions and educational benefits and difficulties encountered.

KEYWORDS: *The Simpsons*, pop culture, reducing math anxiety, Flatland, writing assignments.

INTRODUCTION

Now in its fifteenth season, *The Simpsons* [10] is an award-winning international pop culture phenomenon. Though it is animated, *The Simpsons* is one of the most literate television programs on the air, containing many references to subject matter and scholars from various academic fields, including mathematics. It is the longest-running sitcom of all time, with original episodes airing Sundays at 8pm (7pm Central) on the Fox television network. In addition, episodes air in syndication daily and some are available on DVD [10].

Since *The Simpsons* has been airing in prime time for most of our students' lives, they are likely familiar with the program and its large cast of characters, including a resident mathematician, Professor Frink. *The Simpsons* also contains over one hundred instances of mathematics ranging from arithmetic to geometry to calculus. While many of these instances are designed to expose and poke fun at public innumeracy, several episodes of *The Simpsons* contain significant mathematics that relates to material we normally cover in our classes. In fact, some of the writers, including Al Jean, Executive Producer and head writer, have degrees in mathematics [6]. For these reasons, this program is an ideal source of fun ways to introduce important concepts to students, and to reduce math anxiety and motivate students in courses for non-majors. Here we explore some of these and discuss how we have used them in the classroom [7].

EXCERPTS AND ACTIVITIES

1. $r \, dr \, r$

In the episode *Bart the Genius* [10, episode 7G02], fourth-grader Bart Simpson cheats on a mathematics test, and after receiving his high score, he transfers to a school for gifted children. His new teacher works out a problem on the board, saying,

Teacher: So $y = \frac{r^3}{3}$. And if you determine the rate of change of this curve correctly, I think you will be pleasantly surprised.

[After a brief moment most of the children laugh appreciatively. The teacher observes that Bart appears confused.]

Teacher: Well, don't you get it, Bart? Derivative $dy = \frac{3r^2 dr}{3}$, or $r^2 dr$, or $r \, dr \, r$. Har de har har! Get it?

[She writes on the board:]

$$y = \frac{r^3}{3}$$

$$dy = \frac{3r^2 dr}{3}$$

$$r^2 dr$$

$$r \, dr \, r$$

This calculus joke is the source of the title of this article.

Exercise 1. Watch the show excerpt. Explain in your own words what the calculus problem is that the teacher is solving. Explain each step of her

written solution. Is it correct? If there is an error, explain and make the correction. Is her verbal explanation completely correct? If not, explain how you would change her wording.

Exercise 2. Explain the joke. If English is not your first language, then you may wish to ask a native English speaker why this problem/solution is funny.

We give students in a first semester course in calculus this assignment as soon as they have learned about derivatives, differentials, and the chain rule. It is a nice way for students to be creative while reviewing important topics just prior to an exam. Asking students to identify errors in proposed solutions encourages them to be skeptical mathematicians, as they should be. Students are uniformly impressed that a joke requiring knowledge of calculus appeared in the first season of a decade-old cartoon. Many of them who are fans of the show have seen this scene numerous times, and since they have just learned the related material, finally understand the joke. One student explained the joke by saying that “har de har har” is what old people say when they laugh. In fact there is some merit to this answer, since that phrase is attributed to Jackie Gleason on the 1950s television program *The Honeymooners*.

2. Pythagorean Theorem

In the episode *Springfield (Or, How I Learned to Stop Worrying and Love Legalized Gambling)* [10, episode 1F08], Homer Simpson retrieves a pair of eyeglasses from a toilet in a men’s restroom. After putting them on, he begins reciting from memory a familiar-sounding result, and is corrected by someone nearby. Here is their exchange:

Homer: The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side.

Man in stall: That’s a right triangle, you idiot!

Homer: D’oh!

Homer’s recitation is identical to what the Scarecrow says upon receiving his “brain” (a diploma) from The Wizard in the 1939 film *The Wizard Of Oz* [13]. (In the original book by L. Frank Baum, his “brain” is a collection of pins and needles to keep him “sharp.”)

Exercise 1. What are Homer and the Scarecrow attempting to recite? Identify all the errors in their version. Find any and all triangles for which

their statement is true.

Exercise 2. Is the correction from the man in the stall sufficient? Give a complete, correct statement of what Homer and the Scarecrow are trying to recite. Do this first using only English words, and a second time using mathematical notation. Use complete sentences.

Exercise 3. Think of another mathematical formula that you know well. As before, give a complete statement of the mathematical result that is encapsulated by the formula. Write it first using only English words, and then again using mathematical notation.

This is an excellent exercise we have used on the first day of a precalculus course, as a fun and nonstressful way to break the ice with students, while reviewing prerequisite geometry. Students at this level must stop simply memorizing formulas and begin to understand what they think they know by rote. Simple geometry and the triangle inequality reveal that the Scarecrow's statement is true for no triangles.

3. 3-D Homer

In the Halloween episode *Treehouse of Horror VI* [10, episode 3F04], one of the three short stories is titled *Homer*³. This 7.5 minute segment is a parody of a classic episode of the television program *The Twilight Zone* in which a girl disappears into another dimension after rolling under her bed.

Homer wants to avoid a visit from his sisters-in-law, and quickly looks for a place to hide. He checks underneath the rug, which appears to lie flat on the floor, but his cat and dog have already claimed that spot for themselves. He notices that he can put his hand through a wall in the living room, and decides to risk it and walk through the wall. In a world resembling that from the movie *Tron*, Homer's body and other objects are rendered as 3-dimensional objects, at least more so than they usually are in this cartoon.

Homer: What's going on here? I'm so bulgy. [He pokes his belly which ripples in response.] My stomach sticks way out in front and my [examines his bulging backside] – aah!

[Family and friends, including Professor Frink and Police Chief Wiggum, arrive to help locate Homer.]

Lisa Simpson: Well, where's my dad?

Frink: Well, it should be obvious to even the most dimwitted individual who holds an advanced degree in hyperbolic topology that Homer Simpson has stumbled into... the third dimension! Here [drawing on a blackboard] is an ordinary square. . .

Wiggum: Whoa, whoa - slow down, egghead!

Frink: . . . but suppose we extend the square beyond the two dimensions of our universe, along the hypothetical z -axis, there.

Everyone: [gasps]

Frink: This forms a three-dimensional object known as a “cube,” or a “Frinkahedron” in honor of its discoverer.

Homer’s voice: Help me! Are you helping me, or are you going on and on?

Frink: Oh, right. And, of course, within, we find the doomed individual. [He draws Homer’s head inside the cube on the board.]

After viewing the segment, without even being prompted, students immediately question whether the Simpsons live in two or three dimensions. Hence this is a great way to motivate students to explore two, three and four physical dimensions and to learn about the shape of space [5]. Here we discuss two sets of exercises on this material.

Exercise 1a. Discuss the differences between 2-D and 3-D Homer. Be sure to relate shadowing, lighting and physical differences in the 2-D and 3-D worlds to an explanation of how these differences give the appearance of added depth.

Exercise 1b. In the supposedly 2-D world, what aspects could be used to argue that the Simpsons are living in a 2-D world?

Exercise 1c. In the supposedly 2-D world, what aspects could be used to argue that they are living in a 3-D world? (Hint: think about their movements and behaviors, the world that is represented, and the use of perspective in the bookcase and in the cube that Professor Frink drew.)

Students at all levels ranging from an introduction to mathematics class for non-majors to an upper division applications of geometry seminar benefit from this assignment as it forces them to examine their own beliefs about dimension.

Creative student responses to Exercise 1b include: the fact that Professor Frink says they are 2-D; the rest of the crowd is surprised by the idea of a third dimension; there are no shadows in the supposedly 2-D part of the segment; and the lighting always stays the same. There are also some subtle points related to this question. The fact that the cat and dog fit underneath the rug indicates that they are flat, but the fact that the rug is on top of them indicates a third dimension. In addition, some students respond with the argument that the Simpsons must be 2-dimensional because they fit on a TV screen. When we remind them that television programs and movies show 2-dimensional representations of 3-dimensional objects all the time, they re-examine their ideas.

Exercise 2. Assume that the Simpsons really are 2-D creatures living in an x - y plane and that Homer is transformed into a 3-D creature. Though a 2-D Marge Simpson couldn't really understand the third dimension and would feel there isn't any room for another dimension, she could see weird behavior occurring that suggests the third dimension exists (for example the "wall" into which Homer disappeared). 2-D Marge wouldn't be able to comprehend the concept of depth or an entire 3-D Homer, since only 2-D pieces would make sense to her. Write a letter from 3-D Homer Simpson to his 2-D wife Marge which discusses his change from 2-D into 3-D. Be sure to include mathematical explanations in addition to physical descriptions.

Students find this extremely challenging but fun. This assignment works well in a seminar on the applications of geometry, although it is too challenging for beginning mathematics students. One difficulty with this assignment is that students may concentrate on creative ideas that are mathematically invalid since we have not yet covered the material in class. For example, one student wrote that 3-D Homer explained to 2-D Marge that he was flying.

Since this assignment is designed to stimulate both mathematical exploration and creative writing, we would rather respond to a student with the comment "I don't understand" than a comment that discourages the creative process. We encourage students to correct their mistakes themselves. Since this writing assignment occurs in a course designated as "writing intensive," the students revise their own work. We rarely directly tell students that their ideas are incorrect, since we want them to explore their own ideas instead of trying to guess at which answer we might expect. In addition, ideas that seem invalid at first are often fine once the students have refined, explained, and clarified their thoughts.

A computer lab on 2-D universes [7] works well at various levels. After some Web readings on 2-D creatures, students explore what it is like to be

a 2-D creature with help from Davide Cervone's movies of a cube passing through Flatland [1, 2]. In order to reinforce the material, students must decide what 2-D Marge would see when a cube passes through her plane of existence at various angles, how 2-D Marge could pass 2-D Lisa, and what 2-D Marge would have to look like in order to eat or see. While students may have problems relating to the idea of a Flatlander, our students are familiar with and amused by *The Simpsons*, and so they identify easily with 2-D Marge. The lab concludes with a discussion of some of the possibilities for the shape of a 2-D universe and related activities such as Jeff Weeks' torus and Klein bottle Tic-Tac-Toe games [11].

During the next lab, usually 1-2 weeks later, students imagine that the Simpsons had been 3-D all along and that Homer had transformed to become a 4th spatial dimensional creature. They complete Web readings [5] and answer questions on the shape and geometry of our universe, the fourth physical dimension, and related real-life applications. For example, they explore what one layer of Homer's skin might resemble after he gains a physical dimension. Students find this lab creative, challenging, and rewarding.

Typically a course segment on the geometry of the universe begins with a discussion of Flatlanders. The advantage of beginning with *The Simpsons* instead is that students relate to them and find them amusing. This helps students feel more comfortable with challenging material. For example, 2-D Marge feels less abstract to students than a Flatlander even though we explore 2-D Marge's world in exactly the same way that we would have explored Flatland in the classroom. After watching the movie clip, without even being prompted, students immediately argue whether the Simpsons are 2-D or 3-D. The clip engages students and encourages them to be creative and hence is invaluable as a pedagogical tool.

The Simpsons theme is woven throughout the shape of the universe segment. This helps motivate students and connects them to what they have previously learned. This is especially important for this topic since students must use their ideas of 2-D behavior in order to understand the 4th dimension and the possibilities for the shape of the universe. While teachers normally direct students to keep Flatlanders in mind while they explore 4-D, this explicit direction is not needed when *The Simpsons* is used. Students have already debated whether Marge is 2-D or 3-D. Hence, they can just as easily imagine 3-D Marge struggling with the concept of 4-D Homer and so the leap to 4-D is quite natural in this setting. For these reasons, *The Simpsons* is a great way to introduce a segment on the shape of space.

4. Fermat's Last Theorem

Even the small details of Homer's new dimension [10, episode 3F04] are mathematically interesting. One of the equations rushing past Homer is

$$1782^{12} + 1841^{12} = 1922^{12}.$$

In a computer lab for an introduction to mathematics class for non-majors [7], students use Web-searching skills in order to find this equation within the text transcript for the episode [8]. Students must then explain why the equation cannot hold. The fact that the left hand side of the equation is odd while the right hand side of the equation is even provides an explanation that is accessible to beginning mathematics students. Next, students search the Web for the statement of Fermat's Last Theorem and relate this statement to the validity of the equation. Students then work on the following exercise.

Exercise. Suppose that you decide to calculate the 12th root of the left hand side of the equation. On a TI-83 calculator, type

$$1782^{12} + 1841^{12} \text{ ENTER}$$

and obtain 2.541210259 E 39. Then take the 12th root by typing

$$\wedge(1/12) \text{ ENTER}$$

and indeed you will obtain 1922. How do you resolve this apparent conflict with our earlier work?

Some students insist that Andrew Wiles [12] and Fermat must have been wrong. Other students correctly recognize the limitations of their calculators. This activity highlights the dangers of blindly accepting results on a calculator and is also a fun way to introduce or review Fermat's Last Theorem.

Elkies [3, 4] refers to these types of equations as Fermat near-misses, that is, equations that nearly contradict Fermat's Last Theorem. Students can use a computer algebra system to examine how close this equation is to being true. Advanced students studying Fermat's Last Theorem can connect this equation to topics in algebraic geometry.

5. π

In the episode *Lisa's Sax* [10, episode 3G02], two young girls at a gifted school play patty-cake while chanting:

Cross my heart and hope to die.

Here's the digits that make π :

3.14159265358979323846...

We ask students in a survey course for non-majors whether the patty-cake game would ever end. Some students say that the game does end, since when they enter π into a calculator they see that the digits stop. This activity is a good starting point to discuss the irrationality of π and the fact that not only would the patty-cake game never end, but it would also never get repetitive since the decimal expansion of π never repeats. Of course, this depends on the mathematical definition of "repetitive," as this game would certainly get boring! This catchy chant helps students memorize digits of π . In addition, both visual and auditory learners benefit from this creative introduction to π . Visual learners picture the girls playing this game that never ends or repeats, while auditory learners remember the chant.

This clip can also motivate students to explore the concept of proof. An introduction to mathematics class asked how mathematicians knew that π was irrational. We discussed the fact that a mathematical proof would have to give a logical demonstration of the irrationality of π without writing down all of the digits. While we told the class that proofs using calculus did exist but were beyond the scope of the course, one student approached us after class and asked us to e-mail the proof to her anyway. The girls' game also suggests natural questions about how often individual digits appear in the number's expansion, which can lead to discussions on the normality of π .

CONCLUSION

Thanks to the program's longevity, popularity and cleverness, and its vast quantity of mathematical moments, studying *The Simpsons* is a wonderful way to have fun in class while introducing or examining important topics. Students begin these exercises by laughing at the jokes and familiar characters. Their laughter and enjoyment alleviate any math anxiety, and pave the way for a serious study of the mathematics involved.

Over a period of several years, only one of our hundreds of students commented on an evaluation that using cartoons was "stupid." Most students find it interesting, entertaining, and creative. Recently a student told us that the cartoons were great and fun ways to get them excited about the mathematics. Our favorite comment comes from a college administrator who admitted that while she had always been math-phobic, a talk on our Simpsons work made her want to study more mathematics. We love this

comment since that is exactly the reason that we use these clips, and this is the response we hope for in our own students.

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BIOGRAPHICAL SKETCHES

Sarah J. Greenwald is an assistant professor of mathematics at Appalachian State University in Boone, NC. She received her PhD in 1998 from the University of Pennsylvania and received her BS from Union College in Schenectady NY in 1991. Her research area is the Riemannian geometry of orbifolds and she is also interested in the geometry of the universe. She enjoys creating classroom activity sheets in order to engage students with mathematics related to *The Simpsons* and has taught a class on women and minorities in mathematics with portions dedicated to both mathematical content and equity issues.

Andrew Nestler is an associate professor of mathematics at Santa Monica College, one of the nation's largest community colleges. He received his BA from Pomona College in 1991, his MA from the University of Pennsylvania in 1993, and his PhD from the University of Southern California in 2000. His dissertation topic was the algebraic K-theory of curves over finite fields. At SMC he concentrates on teaching and advising a wide variety of students.