Application of Algebra: Periodic Payment

\$100 every month for 25 years, compounded monthly at 5%?
deposit money at the end of each compounding period

http://image.naldzgraphics.net/2012/10/8-periodic-pay.jpg

- future value of each payment via lump sum:
  1st payment of \$100 grows to

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- future value of each payment via lump sum:
  1st payment of $100 grows to $100(1 + \frac{.05}{12})^{299}$

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- future value of each payment via lump sum:
  1st payment of \$100 grows to $100(1+\frac{.05}{12})^{299}$
  2nd payment of \$100 grows to

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- future value of each payment via lump sum:
  1st payment of \$100 grows to $100(1+\frac{.05}{12})^{299}$
  2nd payment of \$100 grows to $100(1+\frac{.05}{12})^{298}$
  ...
  299th payment of \$100 grows to

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  ...
  299th payment of $100 grows to $100(1 + \frac{.05}{12})^{1}$
  300th payment of $100 grows to $100(1 + \frac{.05}{12})^{0} = 100$

Periodic Payment: It's all about the Benjamins

\$100 every month for 25 years, compounded monthly at 5%?

- total savings plus interest (FV for short on these slides) sum the future value of each payment:
  $FV=100(1+\frac{.05}{12})^{299}+100(1+\frac{.05}{12})^{298}+\ldots+100(1+\frac{.05}{12})^{1}+100$

"an investment in knowledge pays the best interest"

Image Credit: Dan Rosandich

- FV=$100(1+\frac{.05}{12})^{299}+100(1+\frac{.05}{12})^{298}+\ldots+100(1+\frac{.05}{12})^{1}+100$
- too many terms—as many as compounding periods!
- shift our view via algebraic argumentation as in the reading
- FV(1+rate) = FV$(1+\frac{.05}{12})$...

Mat Moore, 2008

1. $FV=100(1+\frac{.05}{12})^{299}+100(1+\frac{.05}{12})^{298}+\ldots+100(1+\frac{.05}{12})^{1}+100$
2. FV(1+rate) = $FV(1+\frac{.05}{12})$ =
   $[100(1+\frac{.05}{12})^{299}+100(1+\frac{.05}{12})^{298}+\ldots+100(1+\frac{.05}{12})^{1}+100](1+\frac{.05}{12})$

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   FV$(1+\frac{.05}{12})$ =
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4. adjust the power

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5. subtract 2 equations [line 4-line 1]—cancel overlap to simplify
   $FV(1+\frac{.05}{12})-FV$ =
   $100(1+\frac{.05}{12})^{300}+100(1+\frac{.05}{12})^{299}\ldots+100(1+\frac{.05}{12})^{2}+100(1+\frac{.05}{12})$
   $-$
   $100(1+\frac{.05}{12})^{299}+100(1+\frac{.05}{12})^{298}+\ldots+100(1+\frac{.05}{12})^{1}+100$
6. what is left?

1. $FV = 100(1+\frac{.05}{12})^{299} + 100(1+\frac{.05}{12})^{298} + \ldots + 100(1+\frac{.05}{12})^{1} + 100$
2. FV(1+rate) = $FV(1+\frac{.05}{12}) =$
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   $FV(1+\frac{.05}{12}) - FV =$
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   FV$(1+\frac{.05}{12})-$ FV = $100(1+\frac{.05}{12})^{300}-100$
7. a bit more algebra: factor 100 and FV and solve for FV:
   FV $(1+\frac{.05}{12}-1)=100((1+\frac{.05}{12})^{300}-1)$

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   $FV\,(1+\frac{.05}{12} - 1) = 100((1+\frac{.05}{12})^{300} - 1)$

$$\text{total savings + interest} = \frac{100((1+\frac{.05}{12})^{300} - 1)}{\frac{.05}{12}}$$

periodic payment: total savings + interest $= \frac{100((1+\frac{.05}{12})^{300}-1)}{\frac{.05}{12}}$

- sum the future value of each payment
- too many terms—as many as compounding periods!
- shift our view via transforming it by a common piece (1+rate) and then we combined the shifted equation with the original (subtraction). The overlap cancelled to give us a manageable formula
- analogy: Jeff Weeks cannot view higher dimensional objects directly, so instead he pieces together lower dimensional objects that are viewable and manageable

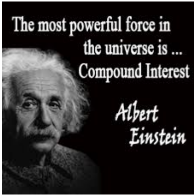

Image credits: Inkley, http://investorhorizon.com/wp-content/uploads/2015/02/ci7.jpg

periodic payment

$100 every month for 25 years, compounded monthly at 5%?

total savings + interest =

$$\frac{\text{regular payment}((1 + \text{periodic rate})^{\text{\# times compounded}} - 1)}{\text{periodic rate}}$$

$$\frac{100((1 + \frac{.05}{12})^{300} - 1)}{\frac{.05}{12}}$$

$$\frac{100((1 + \frac{.05}{12})^{300} - 1)}{(\frac{.05}{12})}$$

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= \$59,550.97
total interest?

periodic payment

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total interest? total savings plus interest - amount we put in

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$$\frac{100((1 + \frac{.05}{12})^{300} - 1)}{(\frac{.05}{12})}$$

= \$59,550.97
total interest? total savings plus interest - amount we put in
$= \$59,550.97 - 100 \times 12 \times 25 = \$29,550.97$
$= \$59,550.97 - 100 \times 300 = \$29,550.97$