The Order of a Difference Equation and Initial Conditions

Suppose we wanted to know $Y_7$. From the rule we can see,

$$Y_7 = Y_6 + Y_5 = 8 + 5 = 13$$

This comes easily because we have already generated the sequence. Pretend now that we do not have the sequence calculated for us and all we have is $Y_{k+2} = Y_{k+1} + Y_k$. Now determining $Y_7$ becomes rather challenging because we have no way of knowing the values of $Y_6$ or $Y_5$. To help with this dilemma we will look back at the generalized difference equation of order $n$, $Y_{k+n} = F(k, Y_{k+n-1}, Y_{k+n-2}, Y_k)$ in comparison to the Fibonacci equation and provide a couple more definitions.

In the Fibonacci difference equation, $n=2$ and the order is 2. The order of a difference equation is defined by the difference between the highest and the lowest terms in the equation. The highest term in the Fibonacci equation is $Y_{k+2}$ and the lowest is $Y_k$. $k+2 - k = 2$ thus the order is 2. This is how difference equations get their name. The terms in the equation are discrete differences. In general the highest term is $k+n$ and the lowest term is $k$. $k+n - k = n$ thus $n$ is the order of the generalized equation. The order helps to determine the number of initial conditions needed to lock down a sequence. If the order of the difference equation is $n$, then $n$ initial conditions are needed to make the sequence unique. The Fibonacci equation needs 2 initial conditions. The first member determined by the equation is $Y_2$ where $k=0$. Therefore we need to define $Y_1$ and $Y_0$. With the initial conditions $Y_1 = 1$ and $Y_0 = 0$, the Fibonacci sequence is complete and unique.