To derive the Cartesian expression to the Witch of Agnesi, we refer to the picture above with certain stipulations, which are given below:

1.) $AQ \perp OQ$
2.) $AP \perp AQ$
3.) $AP \perp BP$
4.) $\Delta AQO$ is similar to $\Delta BPA$

By looking at these triangles above, we can see that the right triangles $\Delta AQO$ and $\Delta ABQ$ are similar because they share a common acute angle. $\Delta QBO$ and $\Delta AQO$ are similar. Now we can get the equation:

$$\frac{AQ}{QQ} = \frac{BO}{BP} = \frac{PA}{PA}$$

If we label $B=(u,v)$ and $P=(x,y)$, we can use substitution in the previous equation and get:
\[
\frac{x}{x-u} = \frac{a}{a-y}
\]

We then cross-multiply and get:

\[x(a-y) = a(x-u)\]

After solving the equation for \(u\), the x-coordinate of \(B\), we get:

\[u = \frac{xy}{a}\]

By using the Pythagorean Theorem, \(a^2 + b^2 = c^2\), where \(a\) and \(b\) are the sides of a right triangle and \(c\) is the hypotenuse, we can derive the following where \(m\) is the line parallel to \(y=2\) that goes through the point \((0,1)\):

\[
\text{in } \triangle OBK, \quad OB^2 = u^2 + y^2
\]

\[
\text{in } \triangle QMB, \quad BQ^2 = (a-v)^2 + u^2
\]

\[
\text{in } \triangle QOB, \quad a^2 = OB^2 + BQ^2
\]

By using all of the above equations above and using substitution, we get:

\[a^2 = (u^2 + y^2) + ((a-y)^2 + u^2)\]

Simplifying the above equation we get \(u^2 + y^2 - ay = 0\)

We already know that \(u = \frac{xy}{a}\), substitute this into the equation and solve for \(y\) and \(A\)

We get the Cartesian expression for the Witch of Agnesi:

\[y = \frac{a^3}{x^2 + a^2}\]