Question 1: If we put in $100 now and leave it there for 25 years compounded monthly at 5% how much will we end up with? How much of that will be interest ($)?

Question 2: If we put in $100 each month for 25 years compounded monthly at 5% how much will we end up with? How much will be interest ($)?
Question 2:
Put in $100/month * 12 months/year * 25 years = $30,000, so we will end up with $30,000 + interest ($) = ?

Not a lump sum because we put in money each month – start with what we know and try to use it to develop a new formula.

25 years from now:
Our 1st payment of $100 will have grown to $100(1+\frac{.05}{12})^{299}$
Our 2nd payment of $100 will have grown to $100(1+\frac{.05}{12})^{298}$
...
Our 299th payment of $100 will have grown to $100(1+\frac{.05}{12})^{1}$
Our 300th payment of $100 will have grown to $100(1+\frac{.05}{12})^{0} = 100$

Note: Banks set it up so that we deposit money at the end of each month, so the money gains interest 1 month less than we expect (for example, 299 = 25*12-1=300-1).

Equation 1: Total Savings Plus Interest =
$100(1+\frac{.05}{12})^{299} + 100(1+\frac{.05}{12})^{298} + 100(1+\frac{.05}{12})^{297} + \ldots + 100(1+\frac{.05}{12})^{1} + 100$

Each term in Equation 1 is the future value of one payment we make using the lump sum formula. But, Equation 1 is too complicated to use for many months or years. We’ll step back and shift our view of it. We’ll transform it by a common piece and then combine the shifted equation with the original. The overlap will cancel so that we can obtain a general formula to use. This is not just a trick – it is a different way of looking at things.
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Shifted Equation:
Equation 1 * (1+\frac{.05}{12}) = Total Savings Plus Interest* (1+\frac{.05}{12}) =
\[\left[100(1+\frac{.05}{12})^{299} + 100(1+\frac{.05}{12})^{298} + \ldots + 100(1+\frac{.05}{12})^{1} + 100\right] * (1+\frac{.05}{12}) =

Now distribute the (1+\frac{.05}{12}) term inside the square brackets

=\left[100(1+\frac{.05}{12})^{299}(1+\frac{.05}{12}) + \ldots + 100(1+\frac{.05}{12})^{1}(1+\frac{.05}{12}) + 100(1+\frac{.05}{12})\right]

We combine the (1+\frac{.05}{12}) terms using exponent rules blah^n blah = blah^{n+1}

=\left[100(1+\frac{.05}{12})^{300} + 100(1+\frac{.05}{12})^{299} + \ldots + 100(1+\frac{.05}{12})^{2} + 100(1+\frac{.05}{12})\right]

=Total Savings Plus Interest* (1+\frac{.05}{12})
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**Shifted Equation – Equation 1:**

The left hand sides:

Total Savings Plus Interest* \((1 + \frac{.05}{12})\) - Total Savings Plus Interest

The right hand sides:

\[
[100(1+\frac{.05}{12})^{300} + 100(1+\frac{.05}{12})^{299} + \ldots + 100(1+\frac{.05}{12})^2 + 100(1+\frac{.05}{12})] \\
- [100(1+\frac{.05}{12})^{299} + 100(1+\frac{.05}{12})^{298} + 100(1+\frac{.05}{12})^{297} + \ldots + 100(1+\frac{.05}{12})^1 + 100]
\]

Cancel out common terms

\[= 100(1+\frac{.05}{12})^{300} - 100\]

We now have **Shifted Equation – Equation 1** =

Total Savings Plus Interest* \((1 + \frac{.05}{12})\) - Total Savings Plus Interest

\[= 100(1+\frac{.05}{12})^{300} - 100\]

We want to solve for Total Savings Plus Interest. Factor this out and solve!