1. A problem from prepositional calculus

A new operator, $\oplus$, or exclusive-or may be defined by the following truth table:

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \oplus q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Create a propositional calculus expression using only $\land$, $\lor$ and $\neg$ that is equivalent to $p \oplus q$. Prove their equivalence using truth tables.

2. The if and only if operator

The logical operator $\leftrightarrow$ is read if and only if $p \leftrightarrow q$ is defined as being equivalent to $(p \Rightarrow q) \land (q \Rightarrow p)$. Based on this definition, show that $p \leftrightarrow q$ is logically equivalent to $(p \lor q) \Rightarrow (p \land q)$:

a. By using truth tables.
b. By a series of substitutions using the identities in Figure 2.2.

3. Some unification problems

Attempt to unify the following pairs of expressions. Either show their most general unifiers or explain why they will not unify.

a. $p(X,Y)$ and $p(a,Z)$.
b. $p(X,X)$ and $p(a,b)$.
c. $\text{ancestor}(X,Y)$ and $\text{ancestor}(\text{bill}, \text{father}(\text{bill}))$.
d. $\text{ancestor}(X, \text{father}(X))$ and $\text{ancestor}(\text{david}, \text{george})$.

4. A logical puzzle, you only need to write rules to prove that you are your own grandparent; this makes the problem simpler since you don’t have to deal with gender.

The following story is quoted from N. Wirth’s “Algorithms + data structures = programs” (Wirth 1976).

I married a widow (let’s call her W) who has a grown-up daughter (call her D). My father (F), who visited us quite often, fell in love with my step-daughter and married her. Hence my father became my son-in-law and my step-daughter became my mother. Some months later, my wife gave birth to a son ($S_1$), who became the brother-in-law of my father, as well as my uncle. The wife of my father, that is, my step-daughter, also had a son ($S_2$).

Using predicate calculus, create a set of expressions that represent the situation in the above story. Add expressions defining basic family relationships such as the definition of father-in-law and use modus ponens on this system to prove the conclusion that “I am my own grandfather.”

**BONUS**: Write your rule in prolog using $me$, $w$, $d$, $f$, $s1$, $s2$ as the constant values and show that grandparent($me$, $me$) is true. Some useful predicates might be parent, grandparent, and married.