Lecture 4 Exercises

1. Prove that the interpretation of every System F type is an equality preserving fibred functor.

2. Show that the identity fibration $\mathsf{Id} : \mathsf{Set} \to \mathsf{Set}$ is an instance of our framework, i.e., that $\mathsf{Rel}(\mathsf{Id})$ is an equality preserving arrow fibration that is also a $\forall$-fibration.

3. Show that the identity fibration $\mathsf{Id} : \mathsf{Set} \to \mathsf{Set}$ models *ad hoc* polymorphism by demonstrating that, for example, $[\![\vdash \forall\alpha.\alpha \to \alpha]\!]_o$ contains the non-natural transformation η for which $\eta_{\mathsf{Bool}}(x) = \neg x$ and $\eta_X(x) = x$ for $X \neq \mathsf{Bool}$.

Note that this does not contradict the fact that (ignoring size issues — see Problem 5 below) Reynolds' construction gives an instance of our framework via the relations fibration on Set because $\mathsf{Rel}(\mathsf{Id})$ does not give the relations fibration on Set defined in the previous lecture.

4. [AMBITIOUS] Show that the model of System F constructed in this lecture is actually a $\lambda 2$-fibration. (See Theorem 4.6 of the original MFPS'15 paper — version with the appendix included — for some hints.)

5. [AMBITIOUS] Flesh out the examples given in this lecture. In particular, (ignoring size issues) show that Reynolds' Set construction gives an instance of our framework via the relations fibration on Set.

7. [AMBITIOUS] Phil Wadler popularized Reynolds' parametricity in his 1989 paper, *Theorems for free!*. With respect to the concrete domain-theoretic model of System F that Wadler gives there:

- Unwind the parametricity theorems for the specific types in Figure 1 of that paper.
- Show that the domain-theoretic model given there is an instance of our bifibrational framework.
- Conclude that our model thus validates the usual "free theorems" for System F terms.